Short Communications

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Elastic coefficients of higher orders in crystals.

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A group theoretical method has been used to calculate the number of independent sixth and seventh order elastic coefficients for all 32 crystal classes.

A group theoretical method has been employed in determining the number of independent elastic coefficients of crystals in the 32 classes. Previously, these numbers have been calculated for the third-order coefficients by Jahn (1949) and Bhagavantam & Suryanarayana (1949) independently and for the fourth and fifth-order coefficients by Krishnamurty & Gopalakrishnamurty (1968).

In this note we apply Jahn's (1938, 1949) method to determine the number of independent coefficients for the sixth and seventh order. The results for the 32 crystal classes, as well as for isotropic solids, are obtained.

By use of Tisza's (1933) notation for the symmetrical product, $[\mathbf{V}^2]$, of a polar vector $\mathbf{V}$ with itself, the method of Jahn (1938, 1949) can easily be extended to higher symmetrical powers of a reducible representation. The numbers of independent constants describing the 6th and 7th order elastic constants are then given by the number of times the identical representation occurs in the reduced form of the appropriate representatives $[[\mathbf{V}^2]^6]$ and $[[\mathbf{V}^2]^7]$ respectively. The formulas obtained for the symmetrical products to the 6th and 7th power of reducible representation are:

$$[(\mathbf{D}_0+\mathbf{D}_2)^6]=[\mathbf{D}_2^6]+[\mathbf{D}_2^5]+[\mathbf{D}_2^4]+[\mathbf{D}_2^3]+[\mathbf{D}_2^2]+\mathbf{D}_2+\mathbf{D}_0$$

$$[(\mathbf{D}_0+\mathbf{D}_2)^7]=[\mathbf{D}_2^7]+[\mathbf{D}_2^6]+[\mathbf{D}_2^5]+[\mathbf{D}_2^4]+[\mathbf{D}_2^3]+[\mathbf{D}_2^2]+\mathbf{D}_2+\mathbf{D}_0\,.$$

Using the forms of $[\mathbf{D}_2^n]$ for $n>10$ given by Krishnamurty & Appalanarasimham (1969) the reduced representations for an isotropic solid (group $\mathbf{R}_\infty^i$) are then obtained.

$$[[\mathbf{V}^2]^6]=7\mathbf{D}_0+9\mathbf{D}_2+3\mathbf{D}_3+9\mathbf{D}_4+3\mathbf{D}_5+7\mathbf{D}_6+2\mathbf{D}_7+4\mathbf{D}_8+\mathbf{D}_9+2\mathbf{D}_{10}+\mathbf{D}_{12}$$

$$[[\mathbf{V}^2]^7]=8\mathbf{D}_0+12\mathbf{D}_2+4\mathbf{D}_3+12\mathbf{D}_4+5\mathbf{D}_5+10\mathbf{D}_6+4\mathbf{D}_7+7\mathbf{D}_8+2\mathbf{D}_9+4\mathbf{D}_{10}+\mathbf{D}_{11}+2\mathbf{D}_{12}+\mathbf{D}_{14}\,.$$

One notes that the numbers of independent 6th and 7th elastic constants (*i.e.* the coefficients of $\mathbf{D}_0$) for isotropic solids are 7 and 8 respectively. These results agree very well with those of Krishnamurty & Appalamarasimham (1969). Since each of the 32 crystal point groups is a subgroup of R_∞^i, we can derive the reduced form of the representations for each point group, and based on these forms the number of independent elastic constants for each of the crystal classes is determined. The results for 6th and 7th order constants are given below:

$\bar{1}$, 1:462 (792); m, 2, $2/m$: 246 (416); $2mm$, 222, $2/m\ 2/m\ 2/m$: 138 (228); 4, $4/m$, $\bar{4}$: 124 (208); $4mm$, $\bar{4}2m$, 422, $4/m\ 2/m\ 2/m$: 77 (124); 3, $\bar{3}$: 156 (256); $3m$ 32, $\bar{3}2/m$: 93 (148), $3/m$, 6, $6/m$: 84 (140); $\bar{6}2m$, $6mm$, 622, $6/m\ 2/m\ 2/m$: 57 (90); 23, $2/m\bar{3}$: 48 (76); $\bar{4}3m$, 432, $4/m\ \bar{3}2/m$: 32 (48).

Here the numbers inside the brackets are the appropriate number of coefficients for the 7th order.

We would like to point out that the relationship for the number of independent nth order elastic constants of cubic crystals given by Krishnamurty (1963), *viz.*

$$N=n^2-2n+3$$

is proved not applicable for $n>5$. The complete list of the non-vanishing 6th order elastic coefficients for some crystal classes will be published shortly.

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